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# NLO corrections in MC event generator for angular distribution of Drell-Yan lepton pair production

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Abstract: Using a subtraction method, we derive the formulae suitable for use in Monte-Carlo event generators to give the angular distribution for the gluon-quark induced NLO corrections in Drell-Yan lepton pair production. We also give the corresponding helicity density matrix for $W$ and $Z$ boson production.

Keywords: QCD, NLO Computations, Spin and Polarization Effects.



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# 1. Introduction

In ref. [1] we applied the subtractive method of [2, 3] to derive the NLO gluon-quark-induced contribution for the Drell-Yan process in a form suitable for use in a Monte-Carlo event generator. In this paper we extend these results to provide the angular distribution for the leptons, and, equivalently, the helicity density matrix for $W$ and $Z$ boson production.

As described in [1, 2, 3], an event generator using this subtraction method generates two classes of events: One class is obtained from the leading order (LO) subprocess by showering the initial and final state quarks; the other class is generated by starting with a subtracted next leading order (NLO) subprocess. As in [1], we only treat the gluon-quark induced subprocesses, $qg \to Vq' \to q'll'$ ($V = \gamma^*$, $W^{\pm}$ or $Z^0$). This NLO subprocess is not necessarily suppressed by a factor of $\alpha_s$ if the gluon distribution function is larger than the quark distribution function. The quark-antiquark induced NLO corrections are complicated by the presence of soft divergences [4], so we defer their detailed treatment in our method to the future.

The polarization state of the gauge boson ($\gamma^*$, $W^{\pm}$ or $Z^0$) determines the angular distribution of the decay leptons in the lepton pair rest frame. The general structure of the angular distribution is given by nine helicity cross sections corresponding to the nine helicity density matrix elements for the gauge boson [5]. In the LO parton-level subprocesses ($q\bar{q} \to V \to ll'$) only the transverse polarization of gauge boson contributes to the helicity density matrix. In NLO, in general, all three polarizations of the gauge boson contribute, but the subtraction term, associated with a collinear divergence, only involves the same transverse polarization as in LO. For the $\gamma^*$, only the 4 parity-conserving terms in the cross section are nonzero [6]; but for $W^{\pm}$ and $Z^0$, with their characteristic $V-A$ and $V+A$ couplings, there are two other nonzero helicity cross sections [5].

The organization of this paper is the following. In section 2, we describe the treatment of lepton pair production in an event generator. Then we compute the effect of combining the LO parton-level cross section with the order $\alpha_s$ part of the shower; this will be needed as the subtraction term in the NLO hard cross section calculation. In section 3, we carry out the subtraction from NLO helicity cross sections to get the angular distribution in the NLO differential cross sections. For $W^{\pm}$ and $Z^0$ production, we also give the helicity density matrix. Finally, we summarize and discuss our results in section 4.

## 2. Monte-Carlo algorithms

### 2.1 QCD improved parton model and inclusive lepton pair production

Our calculations are based on the QCD improved parton model, which shows that a hard scattering process initiated by two hadrons is the result of an interaction between their constituents, namely quarks and gluons [7].

We consider the process $A+B \to V+X \to ll' + X$ in a collision of two hadrons $A$ and $B$ at a center-of-mass energy-squared $s=(P_A+P_B)^2$. When the momentum fractions carried by the partons are not too small (roughly $\geq 0.01$), LO subprocesses ($q\overline{q} \to V \to ll'$) are dominant, with the NLO subprocesses being suppress by a factor of $\alpha_s(Q^2)$. But at the higher energies of the Fermilab Tevatron ($\sqrt{s}=1.8\,TeV$), the momentum fractions are often smaller, and the gluon density can be larger than the quark and antiquark densities, so that gluon-induced NLO subprocesses like $qg \to Vq' \to q'll'$ are not necessarily suppressed by $\alpha_s$ relative to the LO subprocess.

The LO cross section $d\sigma^{\rm LO}(V)/dQ^2dyd\Omega$ for producing a lepton pair with rapidity $y$, invariant mass-squared $Q^2$ and relative solid angle $\Omega=(\theta,\phi)$ in the Collins-Soper frame [8] is obtained by weighting the parton level cross section with the parton distribution functions (pdf's) $f_{i/A}(x_a,Q^2)$ and $f_{\overline{\imath}/B}(x_b,Q^2)$ [7]:

$$\begin{aligned} \frac{d\sigma^{\rm LO}(V)}{dQ^2dyd\Omega} &= \sum_{q\overline{q}'} \hat{\sigma}(V)\mathcal{D}_{q\overline{q}'}(\theta) f_{q/A}(x_a,Q^2) f_{\overline{q}'/B}(x_b,Q^2) + \\ &\quad + \sum_{\overline{q}'q} \hat{\sigma}(V)\mathcal{D}_{\overline{q}'q}(\theta) f_{\overline{q}'/A}(x_a,Q^2) f_{q/B}(x_b,Q^2)\,, \end{aligned} \tag{2.1}$$

where

$$\begin{aligned} \mathcal{D}_{q\overline{q}'}(\theta) &= (1+\cos^2\theta + 2A_qA_l\cos\theta)\,, \\ \mathcal{D}_{\overline{q}'q}(\theta) &= \mathcal{D}_{q\overline{q}'}(\pi-\theta)\,. \end{aligned} \tag{2.2}$$

The parton-level cross section $\hat{\sigma}(V)$ and the coefficients $A_q$ and $A_l$ depend on the boson being produced:

- $V = \gamma^*$:

$$\hat{\sigma}(\gamma^*) = \frac{3}{16\pi} \times \frac{4\pi\alpha^2 e_q^2}{9Q^2 s}\,, \qquad A_q = A_l = 0\,, \tag{2.3}$$

where $e_q$ is the quark charge in units of the size of the electron charge $e$. The summation is over $(q, \overline{q}')$ in $\{(u, \overline{u}),\ (d, \overline{d}),\ (s, \overline{s}),\ (c, \overline{c})\}$, where we restrict our attention to quarks that can be approximated as massless in the production of $W$ and $Z$ bosons.

- $V = W^\pm$:

$$\begin{aligned} \hat{\sigma}(W^\pm) &= \frac{3}{16\pi} \times \frac{4\pi\alpha^2}{9s} \frac{|V_{q\overline{q}'}|^2}{16 s_w^4} \times \frac{Q^2}{(Q^2 - M_W^2)^2 + M_W^2 \Gamma_W^2} \\ &\cong \frac{3}{16\pi} \times \frac{\pi}{3}\sqrt{2} G_F |V_{q\overline{q}'}|^2 \frac{Q^2}{s} \delta(Q^2 - M_W^2) B(W \to l\nu) \\ & W^+ : A_q = A_l = 1\,, \qquad W^- : A_q = -A_l = 1\,, \end{aligned} \tag{2.4}$$

where $V_{q\overline{q}'}$ is an CKM matrix element, $s_w = \sin\theta_w$, $G_F$ is the Fermi coupling constant, and $B(W \to l\nu)$ is the branching ratio $\Gamma(W \to l\nu)/\Gamma_W$. In the second line of this equation, we have used the narrow width approximation for the decay of $W$ boson. For the $W^+$, the summation is over the cases $(q, \overline{q}')$ in $\{(u, \overline{d}),\ (c, \overline{s})\}$, while for the $W^-$ we have $(q, \overline{q}')$ in $\{(d, \overline{u}),\ (s, \overline{c})\}$.

- $V = Z^0$:

$$\begin{aligned} \hat{\sigma}(Z^0) &= \frac{3}{16\pi} \times \frac{4\pi\alpha^2}{9s} \frac{[q_V^2 + q_A^2][l_V^2 + l_A^2]}{16 s_w^4 c_w^4} \times \frac{Q^2}{(Q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \\ &\cong \frac{3}{16\pi} \times \frac{\pi}{3}\sqrt{2} G_F [q_V^2 + q_A^2] \frac{Q^2}{s} \delta(Q^2 - M_Z^2) B(Z^0 \to l\bar{l}) \\ & A_q = \frac{2 q_V q_A}{q_V^2 + q_A^2}\,, \qquad A_l = \frac{2 l_V l_A}{l_V^2 + l_A^2}\,, \end{aligned} \tag{2.5}$$

where $q_A = T_q^3$, $q_V = T_q^3 - 2 e_q s_w^2$, $c_w = \cos\theta_w$, $s_w = \sin\theta_w$, $B(Z^0 \to l\bar{l})$ is the branching ratio for $Z^0$, and the summation is the same as in the case of $\gamma^*$. In the second line, we have again used the narrow-width approximation.

The fractional momenta assigned to the incoming quark and antiquark are equal to the following kinematic variables:

$$x_a = e^y \sqrt{\frac{Q^2}{s}}\,, \qquad x_b = e^{-y} \sqrt{\frac{Q^2}{s}}\,. \tag{2.6}$$

With full perturbative QCD corrections taken into account, eq. (2.1) is replaced by the factorization formula,

$$\frac{d\sigma}{dQ^2 dy d\Omega} = \sum_{i,j} \int d\xi_i\, d\xi_j\, f_{i/A}(\xi_i, \mu^2) f_{j/B}(\xi_j, \mu^2) \frac{d\hat{\sigma}_{ij}}{dQ^2 dy d\Omega}\,, \tag{2.7}$$

where the $\xi$'s are the momentum fractions of the incoming partons, and $d\hat{\sigma}_{ij}/dQ^2 dy$ $d\Omega$ is a suitably constructed parton-level hard scattering cross section, which depends on $x_a/\xi_i$, $x_b/\xi_j$, the vector boson mass $Q$, the solid angle $\Omega$, and on the renormalization/factorization scale $\mu$ through $\mu/Q$ and $\alpha_s(\mu)$. Now the sum is over all pairs of parton flavors (quarks and gluons). The formal domain of validity of eq. (2.7) is the asymptotic 'scaling' limit, analogous to the Bjorken limit in DIS, where $s, Q^2 \to \infty$ with $x_a$ and $x_b$ fixed.

### 2.2 Parton-shower algorithm

The initial-state shower algorithm for vector boson production used in PYTHIA or RAPGAP is described in [9] (see also [1, 2]). We use the method described in [1] to define the splitting variable $z$, rather than the $\hat{s}$ method of [9], since the treatment of NLO corrections is simpler.

The part of the algorithm used in an event generator that concerns us is the following:

1. Generate values of $y$ and $Q^2$, then generate a direction (i.e., $\Omega$) for the leptons according to the LO cross section for lepton pair production, eq. (2.1). This gives the initial values of the parton and lepton 4-momenta. (These initial values have zero transverse momentum for the partons and hence for the lepton pair. After showering, the parton 4-momenta will be modified.) The direction $\Omega$ gives polar angles $\theta$ and $\phi$ that we choose to interpret in the Collins-Soper frame [8].

2. For branching on hadron $A$ side: Generate a virtuality $Q_1^2$ for the incoming quark $i$, the first longitudinal momentum splitting variable $z_a$, and an azimuthal angle $\phi'$ for this branching. The distributions arise from the Sudakov form factor

$$S_i(x_a, Q^2_{\max}, Q_1^2) = \exp\left\{ - \int_{Q_1^2}^{Q^2_{\max}} \frac{dQ'^2}{Q'^2} \frac{\alpha_s(Q'^2)}{2\pi} \times \right. \\ \left. \times \sum_k \int_{x_a}^1 \frac{dz_a}{z_a} P_{k \to ij}(z_a) \frac{f_k(x_a/z_a, Q'^2)}{f_i(x_a, Q'^2)} \right\}. \qquad (2.8)$$

   Here, $Q^2_{\max}$ is normally set equal to $M^2$ for $W^\pm$ and $Z^0$, and for $\gamma^*$ it is often fixed to be $Q^2$ — the hard scattering scale. The Sudakov form factor is the probability that the virtuality of quark $i$ is less than $Q_1^2$. The branching on hadron $B$ side is done similarly with first splitting variable $z_b$.

3. Iterate the branching for all initial-state and final-state[1] partons until no further branchings are possible.

4. Compute the actual values of the parton and lepton 4-momenta.

[1] The final-state showering is organized similarly to the initial-state showering. However, we will not need it explicitly in this paper.



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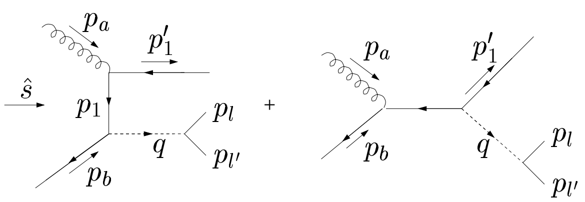


**Figure 1:** NLO gluon-quark collision. Gluon comes from hadron $A$ and (anti)-quark comes from hadron $B$.

### 2.3 Subtraction term from first initial-state branching

In section 3, we will calculate the hard scattering cross section for $gq$-induced process (figure 1). The gluon, of momentum $p_a$, comes from hadron $A$ and the (anti)-quark, of momentum $p_b$, comes from hadron $B$. The NLO contribution we want to calculate is to be accurate when the following conditions are satisfied. 1) The incoming gluon and quark have virtualities and transverse momenta-squared that are small compared with $Q^2$. 2) The intermediate quark, of momentum $p_1$, has a virtuality of order $Q^2$. 3) The lepton pair has a transverse momentum of order $Q$. To avoid double counting, it is necessary to subtract the corresponding contribution obtained from the showering algorithm applied to the LO parton level cross section. It is this subtraction term that we calculate in this section. After this subtraction, the combination of the LO and NLO term will also be accurate when the transverse momentum of the lepton pair or the virtuality of the intermediate quark decreases.

The subtraction term is obtained by multiplying the LO parton level cross section, from eq. (2.1), by the first order term in the expansion of the Sudakov form factor eq. (2.8) in powers of $\alpha_s(Q^2)$. It is made differential in the momenta of the particles involved, and the gluon-to-quark splitting kernel is selected. To match with the definition of the NLO hard-scattering, it should be calculated when the initial-state partons are on-shell and have zero transverse momentum, and the final-state quark is on-shell.

That is, we subtract the first-order cross section in the showering approximation:

$$\frac{d\sigma_{\text{shower 1}}(V)}{dQ^2\,dy\,d\Omega\,dQ_1^2\,d\xi_a\,d\phi'} = \sum_{q,\overline{q}'} \hat{\sigma}(V)\,\mathcal{D}_{q\overline{q}'}(\theta)\frac{\alpha_s(Q^2)}{4\pi^2 Q_1^2}\,C_1(Q_1^2)P(z_a)\,\frac{1}{\xi_a}f_g(\xi_a,Q^2)f_{\overline{q}'}(x_b,Q^2) + \\ +\text{case: antiquarks (from } g\text{) come from } A\text{, quarks from } B. \tag{2.9}$$

Note that in the summation in this equation, we still sum over $q, \overline{q}'$. The gluon from $A$ splits into a $q\overline{q}$ pair, and in the first term the quark from the gluon interacts with the antiquark from $B$. In the second term, on the last line, we have the antiquark from the gluon interacting with the quark from $B$.

The splitting kernel here is for gluon $\to$ quark + antiquark: $P(z) = P_{g\to q\overline{q}}(z) = \frac{1}{2}(1-2z+2z^2)$. Note that because we are doing a strict expansion in powers of $\alpha_s(Q^2)$, the scale argument of the pdf is $Q^2$. The function $C_1(Q_1^2)$ is a cut-off function that gives the maximum value of $Q_1^2$. Normally it is $\theta(Q^2 - Q_1^2)$, but as discussed in [2], a different choice

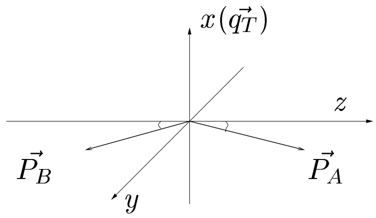


**Figure 2:** Collins-Soper frame. $\vec{P_A}$ and $\vec{P_B}$ lie in the $x$-$z$ plane of the lepton-pair rest frame and z-axis bisects $\vec{P_A}$ and $-\vec{P_B}$.

of the cut-off function could minimize the number of negative weight events generated generated from the subtracted NLO cross section. The general solution to deal with a smooth cut-off function is given in [3] for a non-gauge theory. We plan to apply this method to QCD in the future.

Now, working in Collins-Soper frame [8] (see figure 2), we reconstruct the 4-vectors for the momenta $q$, $p_1'$, $p_a$ and $p_b$ of the lepton pair, the outgoing quark, the incoming gluon and the incoming (anti)-quark. To be consistent with [1], they obey the following requirements:

1. The $z$-axis is given by the Collins-Soper [8] definition for the angles of the leptons; it bisects the angle between $\vec{P}_A$ and $-\vec{P}_B$, where the momenta are for the incoming *hadrons*. This is shown in figure 2. (Bengtsson, Sjöstrand and van Zijl [9] made a different choice of axes in the overall CM frame.)

2. The incoming partons, $p_a$ and $p_b$ have momentum fractions $\xi_a$ and $\xi_b$ relative to their parent hadrons, in the sense of light-front components.

3. $p_1^2 = -Q_1^2$.

4. $p_a^2 = p_b^2 = p_1'^2 = 0$, and $p_a$ and $p_b$ have zero transverse momentum in the overall CM frame.

5. $q^2 = Q^2$, where Q is the invariant mass of the lepton pair.

6. $y = \dfrac{1}{2}\ln\dfrac{P_B \cdot q}{P_A \cdot q} = \dfrac{1}{2}\ln\dfrac{x_a}{x_b}$, is the exact rapidity of the lepton pair.

7. The first splitting variable on the $A$ side is defined by

$$z_a = \frac{x_a}{\xi_a}\,. \tag{2.10}$$

Notice that in this definition, the splitting variable $z_a$ is defined in terms of the kinematic variable $x_a = e^y Q/\sqrt{s}$, rather than in terms of the fractional momentum of the intermediate quark.

We define the Mandelstam variables at the parton level as usual:

$$\hat{s} = (p_a + p_b)^2 = \xi_a \xi_b s \tag{2.11}$$

$$\hat{t} = (p_b - q)^2 = \xi_a x_a \frac{x_b^2 - \xi_b^2}{x_a \xi_b + x_b \xi_a} s = -Q_1^2 \tag{2.12}$$

$$\hat{u} = (p_a - q)^2 = \xi_b x_b \frac{x_a^2 - \xi_a^2}{x_a \xi_b + x_b \xi_a} s \,. \tag{2.13}$$

These satisfy the constraint that $\hat{s} + \hat{t} + \hat{u} - Q^2 = 0$ . Then the partons' 4-momenta in Collins-Soper frame are given by:

$$\begin{aligned}
p_a^\mu &= \xi_a E_A \,(1,\, \sin\eta,\, 0,\, \cos\eta)\,, && (2.14)\\
p_b^\mu &= \xi_b E_B \,(1,\, \sin\eta,\, 0,\, -\cos\eta)\,, && (2.15)\\
q^\mu &= Q\,(1,\, 0,\, 0,\, 0)\,, && (2.16)\\
p_1'^\mu &= p_a^\mu + p_b^\mu - q^\mu \\
&= ([\xi_a E_A + \xi_b E_B - Q],\, [\xi_a E_A + \xi_b E_B]\sin\eta,\, 0,\, [\xi_a E_A - \xi_b E_B]\cos\eta)\,, && (2.17)\\
p_l^\mu &= \frac{Q}{2}\,(1,\, \sin\theta\cos\phi,\, \sin\theta\sin\phi,\, \cos\theta), && (2.18)\\
p_{l'}^\mu &= \frac{Q}{2}\,(1,\, -\sin\theta\cos\phi,\, -\sin\theta\sin\phi,\, -\cos\theta)\,, && (2.19)
\end{aligned}$$

with

$$\begin{aligned}
E_A &= \frac{\sqrt{s}}{2}\sqrt{1+r^2}\,e^{-y}\,, \qquad E_B = \frac{\sqrt{s}}{2}\sqrt{1+r^2}\,e^{+y}\,,\\
\sin\eta &= \frac{-r}{\sqrt{1+r^2}}\,, \qquad \cos\eta = \frac{1}{\sqrt{1+r^2}}\,,\\
r &= \sqrt{\frac{Q_T^2}{Q^2}} = \sqrt{\frac{\hat{u}\hat{t}}{\hat{s}Q^2}}\,.
\end{aligned} \tag{2.20}$$

Convenient variables for the above subtraction term are $Q^2$, $y$, $\Omega$, $\xi_a$ and $Q_1^2$. However, they are not so convenient for the NLO matrix-element calculations. So we transform the cross section in eq. (2.9) in terms of more convenient variables for a hard gluon-quark scattering: $Q^2$, $y$, $\Omega$ , $\xi_a$ and $\xi_b$. Then the cross section after integrating over the branching angle $\phi'$ is:

$$\begin{aligned}
\frac{d\sigma_{\text{shower 1}}(V)}{dQ^2\,dy\,d\Omega\,d\xi_a\,d\xi_b} &= \sum_{q,\overline{q}'} \frac{\alpha_s(Q^2)}{2\pi}\,\hat{\sigma}(V)\,\mathcal{D}_{q\overline{q}'}(\theta) C_1(Q_1^2) P(z_a) \times \\
&\quad\times \frac{2x_b\xi_a\xi_b + x_a(x_b^2+\xi_b^2)}{\xi_a(\xi_b^2 - x_b^2)(x_a\xi_b + x_b\xi_a)} f_g(\xi_a, Q^2) f_{\overline{q}'}(x_b, Q^2) + \\
&\quad+\text{case: antiquarks (from } g\text{) come from } A\text{, quarks from } B\,.
\end{aligned} \tag{2.21}$$

## 3. NLO hard cross section

### 3.1 Unsubtracted NLO term

The angular distribution of the lepton pair produced in Drell-Yan process has been calculated in [5, 10] for large $Q_T$ to order $\alpha_s$. The unsubtracted gluon-quark induced NLO scattering cross section corresponding to figure 1 is:

$$\begin{aligned}
\frac{d\sigma_{\text{unsubtracted }1}(V)}{dQ^2\,dy\,d\Omega\,d\xi_a\,d\xi_b} &= \frac{\alpha_s(Q^2)}{2\pi}\,\frac{s}{\hat{s}}\sum_{q,\overline{q}'}\,\hat{\sigma}(V)\,f_{\overline{q}'}(\xi_b,Q^2)f_g(\xi_a,Q^2)H^{U+L}\times\\
&\quad\times\frac{Q^2(Q^2+\hat{s})}{s^2(x_a\xi_b+x_b\xi_a)^2}\left\{(1+\cos^2\theta)+\sum_{m=0}^{m=4}A_m^{g\overline{q}}(V)G_m(\theta,\,\phi)\right\}\\
&\quad+\text{case: antiquarks (from } g) \text{ come from } A\text{, quarks from } B\,,\\
&= \frac{\alpha_s(Q^2)}{2\pi}\sum_{q,\overline{q}'}\,\hat{\sigma}(V)\,f_{\overline{q}'}(\xi_b,Q^2)f_g(\xi_a,Q^2)\times\\
&\quad\times\left\{-\frac{G^c(\xi_a,\,\xi_b)}{x_b-\xi_b}+H^c(\xi_a,\xi_b\right\}\times\\
&\quad\times\left\{(1+\cos^2\theta)+\sum_{m=0}^{m=4}A_m^{g\overline{q}}(V)G_m(\theta,\,\phi)\right\}\\
&\quad+\text{case: antiquarks (from } g) \text{ come from } A\text{, quarks from } B\,, \qquad (3.1)
\end{aligned}$$

where

$$H^{U+L} = \frac{(Q^2-\hat{s})^2+(Q^2-\hat{t})^2}{-\hat{s}\hat{t}}\,, \qquad (3.2)$$

$$\begin{aligned}
A_0^{gq}(V) &= 4A_1^{gq}(V) = A_0^{g\overline{q}}(V) = 4A_1^{g\overline{q}}(V)\\
&= \frac{\hat{t}\hat{u}[(Q^2+\hat{s})^2+(Q^2-\hat{t})^2]}{(Q^2-\hat{t})(Q^2-\hat{u})[(Q^2-\hat{s})^2+(Q^2-\hat{t})^2]}\,, \qquad (3.3)
\end{aligned}$$

$$A_2^{gq}(V) = A_2^{g\overline{q}}(V) = \frac{1}{2\sqrt{2}}\frac{\sqrt{Q^2\hat{s}\hat{t}\hat{u}}[(Q^2-\hat{u})^2-2(Q^2-\hat{t})^2]}{(Q^2-\hat{t})(Q^2-\hat{u})[(Q^2-\hat{s})^2+(Q^2-\hat{t})^2]}\,, \qquad (3.4)$$

$$A_3^{gq}(V) = -A_3^{g\overline{q}} = A_qA_l\sqrt{\frac{\hat{u}\hat{t}}{8(Q^2-\hat{t})(Q^2-\hat{u})}}\frac{Q^4+\hat{u}^2+2\hat{s}\hat{u}}{(Q^2-\hat{s})^2+(Q^2-\hat{t})^2}\,, \qquad (3.5)$$

$$\begin{aligned}
A_4^{gq}(V) &= -A_4^{g\overline{q}}(V) = A_qA_l\sqrt{\frac{Q^2\hat{s}}{(Q^2-\hat{u})(Q^2-\hat{t})}}\frac{Q^4+\hat{u}^2-2Q^2\hat{t}}{(Q^2-\hat{s})^2+(Q^2-\hat{t})^2}\\
&= A_qA_l(1+\mathcal{A}^{gq}) \qquad (3.6)
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}^{gq} = \mathcal{A}^{g\overline{q}} &= \sqrt{\frac{Q^2\hat{s}}{(Q^2-\hat{t})(Q^2-\hat{u})}}-1-\\
&\quad-\sqrt{\frac{Q^2\hat{s}}{(Q^2-\hat{t})(Q^2-\hat{u})}}\frac{2\hat{t}(\hat{u}+\hat{t})}{(Q^2-\hat{s})^2+(Q^2-\hat{t})^2}\,, \qquad (3.7)
\end{aligned}$$

$$G_0(\theta,\,\phi) = \frac{1}{2}(1-3\cos^2\theta)\,, \qquad (3.8)$$

$$G_1(\theta,\,\phi) = 2\sin^2\theta\cos 2\phi\,, \qquad (3.9)$$

$$G_2(\theta,\,\phi) = 2\sqrt{2}\sin 2\theta\cos\phi\,, \qquad (3.10)$$

$$G_3(\theta,\,\phi) = 4\sqrt{2}\sin\theta\cos\phi\,, \qquad (3.11)$$

$$G_4(\theta,\,\phi) = 2\cos\theta\,, \qquad (3.12)$$

with $\hat{s}$, $\hat{t}$ and $\hat{u}$ being given by eqs. (2.11)–(2.13). Also:

$$H^c(\xi_a,\,\xi_b) = \frac{x_a\xi_a\xi_b^2 + x_a x_b(2x_a\xi_b + x_b\xi_a)}{\xi_a^2\xi_b^2(x_a\xi_b + x_b\xi_a)^3} x_a x_b(x_a x_b + \xi_a\xi_b)\,, \tag{3.13}$$

$$G^c(\xi_a,\,\xi_b) = \frac{x_b(x_a x_b + \xi_a\xi_b)[(x_a x_b)^2 + (x_a x_b - \xi_a\xi_b)^2]}{\xi_a^3\xi_b^2(x_a\xi_b + x_b\xi_a)(x_b + \xi_b)}\,, \tag{3.14}$$

as defined in [11] and used in [1].

Notice that, for $V = \gamma^*$, $A_q = A_l = 0$, so $A_3^{gq}(\gamma^*) = A_4^{gq}(\gamma^*) = 0$, that is, only the first 4 helicity structure functions are nonzero for $\gamma^*$.

In the above eqs. (3.3)–(3.6), only the term containing $A_4^{Gq}(V)$ $(V = W^\pm, Z^0)$ will give a collinear divergence. To get the hard cross section, we need to cancel out the divergence using the subtraction term obtained in eq. (2.21), so we rewrite our NLO unsubtracted parton level cross section eq. (3.1) as:

$$\begin{aligned}\frac{d\sigma_{\text{unsubtracted 1}}(V)}{dQ^2\,dy\,d\Omega\,d\xi_a\,d\xi_b} &= \frac{\alpha_s(Q^2)}{2\pi}\sum_{q,\overline{q}'}\hat{\sigma}(V)\,f_{\overline{q}'}(\xi_b,Q^2)f_g(\xi_a,Q^2)\times\\ &\quad\times\left[-\frac{G^c(\xi_a,\,\xi_b)}{x_b-\xi_b} + H^c(\xi_a,\xi_b)\right]\times\\ &\quad\times\left[\mathcal{D}_{q\overline{q}'}(\theta) + \sum_{m=0}^{m=3} A_m^{g\overline{q}}(V)G_m(\theta,\,\phi) + A_qA_l\mathcal{A}^{g\overline{q}}G_4(\theta,\,\phi)\right]+\\ &\quad+\text{case: antiquarks (from } g\text{) come from } A\text{, quarks from } B\,.\end{aligned} \tag{3.15}$$

### 3.2 NLO term with subtraction

We now subtract the showering term, eq. (2.21). Only the term proportional to $\mathcal{D}_{q\overline{q}'}(\theta)$ and $\mathcal{D}_{\overline{q}'q}(\theta)$ needs a subtraction term; all other terms are finite in the collinear limit ($\hat{t} \to 0$). So we have:

$$\begin{aligned}\frac{d\sigma^{(\text{New})}_{\text{hard 1}}(V)}{dQ^2\,dy\,d\Omega\,d\xi_a\,d\xi_b} &= \frac{\alpha_s(Q^2)}{2\pi}\sum_{q,\overline{q}'}\hat{\sigma}(V)\,f_g(\xi_a,Q^2)\times\\ &\quad\times\Bigg\{\Bigg[f_{\overline{q}'}(\xi_b,Q^2)\Bigg(-\frac{G^c(\xi_a,\,\xi_b)}{x_b-\xi_b} + H^c(\xi_a,\xi_b)\Bigg) -\\ &\qquad - f_{\overline{q}'}(x_b,Q^2)C_1(Q_1^2)P(z_a)\frac{2x_b\xi_a\xi_b + x_a(x_b^2+\xi_b^2)}{\xi_a(\xi_b^2-x_b^2)(x_a\xi_b+x_b\xi_a)}\Bigg]\mathcal{D}_{q\overline{q}'}(\theta) +\\ &\qquad + f_{\overline{q}'}(\xi_b,Q^2)\Bigg(-\frac{G^c(\xi_a,\,\xi_b)}{x_b-\xi_b} + H^c(\xi_a,\xi_b)\Bigg)\times\\ &\qquad\times\Bigg(\sum_{m=0}^{m=3} A_m^{g\overline{q}}(V)G_m(\theta,\,\phi) + A_qA_l\mathcal{A}^{g\overline{q}}G_4(\theta,\,\phi)\Bigg)\Bigg\}\\ &\quad+\text{case: antiquarks (from } g\text{) come from } A\text{, quarks from } B\end{aligned} \tag{3.16}$$

$$= \frac{\alpha_s(Q^2)}{2\pi}\sum_{q,\overline{q}'}\hat{\sigma}(V)\,f_g(\xi_a,Q^2)f_{\overline{q}'}(\xi_b,Q^2)\times$$

$$
\begin{aligned}
&\times\Bigg(\hat{H}^U_{g\overline{q}'}(1+\cos^2\theta)+\left\{-\frac{G^c(\xi_a,\,\xi_b)}{x_b-\xi_b}+H^c(\xi_a,\xi_b)\right\}\times\\
&\quad\times\left\{\sum_{m=0}^{m=3}A_m^{g\overline{q}}(V)G_m(\theta,\,\phi)+A_qA_l\times\right.\\
&\qquad\left.\times\left[\mathcal{A}^{gq}+\hat{H}^U_{g\overline{q}'}\left(-\frac{G^c(\xi_a,\,\xi_b)}{x_b-\xi_b}+H^c(\xi_a,\xi_b)\right)\right]G_4(\theta,\,\phi)\right\}\Bigg)+\\
&+\text{case: antiquarks (from } g\text{) come from } A\text{, quarks from } B, \qquad (3.17)
\end{aligned}
$$

where

$$
\begin{aligned}
\hat{H}^U_{g\overline{q}'} &= -\frac{G^c(\xi_a,\,\xi_b)}{x_b-\xi_b}+H^c(\xi_a,\xi_b)-\\
&\quad-\frac{f_{\overline{q}'}(x_b,Q^2)}{f_{\overline{q}'}(\xi_b,Q^2)}C_1(Q_1^2)P(z_a)\,\frac{2x_b\xi_a\xi_b+x_a(x_b^2+\xi_b^2)}{\xi_a(x_b^2-\xi_b^2)(x_a\xi_b+x_b\xi_a)}\,. \qquad (3.18)
\end{aligned}
$$

Observe that, the coefficient of $\mathcal{D}_{q\overline{q}'}(\theta)$ agrees with the results in [1], detailed calculations shows that the pdf used here is the same as in [1] and [2].

### 3.3 Results for $qg$ subprocess

Now we specify the changes needed for the other gluon-quark scattering subprocess, in which gluon comes out of hadron $B$ instead of $A$. The formulae for the subtracted cross section are obtained from eq. (3.16) by making the following substitutions in the right hand side of those equations:

- Case that quark comes from $A$, antiquark from $B$:

$$a\leftrightarrow b\,,\quad 1\leftrightarrow 2\,,\quad A_m^{g\overline{q}}(V)\leftrightarrow A_m^{qg}(V)\,. \qquad (3.19)$$

- Case that antiquark comes from $A$, quark from $B$:

$$a\leftrightarrow b\,,\quad 1\leftrightarrow 2\,,\quad A_m^{gq}(V)\leftrightarrow A_m^{\overline{q}g}(V)\,. \qquad (3.20)$$

Here

$$
\begin{aligned}
A_m^{qg} &= A_m^{g\overline{q}}(\hat{u}\leftrightarrow\hat{t})\,,\quad A_m^{\overline{q}g}=A_m^{gq}(\hat{u}\leftrightarrow\hat{t})\,,\quad (m=0,\,1,\,4)\,;\\
A_n^{qg} &= -A_n^{g\overline{q}}(\hat{u}\leftrightarrow\hat{t})\,,\quad A_n^{\overline{q}g}=-A_n^{gq}(\hat{u}\leftrightarrow\hat{t})\,,\quad (n=2,\,3)\,. \qquad (3.21)
\end{aligned}
$$

The cut-off function $C_1(Q_1^2)$ is replaced by a function $C_2(Q_2^2)$ with the same functional form, and we define

$$-Q_2^2=\hat{u}=sx_b\xi_b\frac{x_a^2-\xi_a^2}{x_a\xi_b+x_b\xi_a}\,,\quad z_b=\frac{x_b}{\xi_b}\,. \qquad (3.22)$$

### 3.4 Helicity density matrix for $W^\pm$ and $Z^0$ resonance

In some cases, we want to separate the contribution of different spin configurations in the cross section, and then the helicity density matrix is very useful. Instead of giving the explicit formula for helicity density matrix, we give a prescription to obtain the helicity density matrix from the helicity cross sections.

In eq. (3.16), inside the curly brackets, let us define $\hat{H}^U$, $\hat{H}^0$, $\hat{H}^1$, $\hat{H}^2$, $\hat{H}^3$ and $\hat{H}^P$ to be the coefficients of the angular distributions $(1+\cos^2\theta)$, $G_0(\theta,\phi)$, $G_1(\theta,\phi)$, $G_2(\theta,\phi)$, $G_3(\theta,\phi)$ and $G_4(\theta,\phi)$. In [5], these are expressed in terms of what are called 'helicity density matrix elements', but the normalization condition of a density matrix, that the trace is unity, is not satisfied. So we will simply called them 'helicity matrix elements'. Thus the $\hat{H}$'s are linear combination of the helicity matrix elements. The helicity matrix $H_{ab}^{\sigma\sigma'}$ is then given by

$$\begin{aligned} H_{ab}^{00} &= \hat{H}_{ab}^{0} \\ H_{ab}^{++} &= (\hat{H}_{ab}^{U} - \hat{H}_{ab}^{0} + \hat{H}_{ab}^{P})/2 \\ H_{ab}^{--} &= (\hat{H}_{ab}^{U} - \hat{H}_{ab}^{0} - \hat{H}_{ab}^{P})/2 \\ H_{ab}^{+-} &= H_{ab}^{-+} = \hat{H}_{ab}^{1} \\ H_{ab}^{+0} &= H_{ab}^{0+} = \hat{H}_{ab}^{3} + \hat{H}_{ab}^{2} \\ H_{ab}^{-0} &= H_{ab}^{0-} = \hat{H}_{ab}^{3} - \hat{H}_{ab}^{2}\,, \end{aligned} \tag{3.23}$$

where the subscript $ab$ labels the parton content of the cross section. Then the actual helicity density matrix is obtained from the above matrix by dividing it by $\hat{H}_{ab}^{U}$.

In the subprocess $gq \to Vq' \to q'll'$, we have $ab = gq$, while in the subprocess $qg \to Vq' \to q'll'$, we have $ab = qg$.

## 4. Conclusion

We have extended the application of Collins's MC algorithm to give the angular distribution of Drell-Yan lepton pair production at order $\alpha_s$. We also gave the corresponding helicity density matrix of $W$ and $Z$ bosons. We confirmed that the pdf's used in Collins algorithm are process-independent at order $\alpha_s$.

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